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PHYSICAL SPECTRA IN STRING THEORIES

— BRST Operators and Similarity Transformations — [1]

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ABSTRACT

Several examples of similarity transformations connecting two string theories with different backgrounds are reviewed. We also discuss general structure behind the similarity transformations from the point of view of the topological conformal algebra and of the non-linear realization of gauge symmetry.

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1 Introduction

In string theories, BRS symmetry plays crucial role especially in defining physical spectrum[1, 2, 3, 4]. Let us recall briefly the basic ingredients for the BRS quantization of, say, bosonic string.

Taking the conformal gauge, the BRS charge in left-moving sector[1] is defined by

$$Q_B = \oint \frac{dz}{2\pi i} \left(cT + bc\partial c \right), \tag{1}$$

where $c(z)$ and $b(z)$ are the reparametrization ghost and anti-ghost respectively, and $T(z)$ is the energy-momentum tensor for the string coordinates (or the matter part, in the world-sheet sense). $T(z)$ satisfies the OPE

$$T(y)T(z) \sim \frac{c/2}{(y-z)^4} + \frac{2}{(y-z)^2} T(z) + \frac{1}{y-z} \partial T(z), \tag{2}$$

where c is the so-called central charge. The nilpotency of the BRS charge $Q_B^2 = 0$ is guaranteed only when $c = 26$[2].

In use of the BRS charge physical states are defined by the Kugo-Ojima[5] condition:

$$Q_B |phys\rangle = 0. \tag{3}$$

Solving this equation we obtain the physical spectrum of the theory as BRS cohomology. For instance, the physical spectrum of the critical bosonic string consists of the DDF[6] states up to BRS exact states[2, 7, 8, 9].

Thus Q_B governs the physical spectrum of the string theory. If the Q_B's are related in some way between two apparently different string theory, then the spectra of these theories should be also related. Actually there exist such cases. In this talk I present some examples in which the two different string theories are related by the similarity transformations. Also I discuss about the meaning of such kind of similarity transformations from the two points of view: one is from topological algebra and the other is from non-linearly realized gauge symmetry.

These observation will be important for the long-standing problem of finding background independent formulation of string theory and prospected universal theory of string. Even if we failed finding them, it is still important for classifying the universality class of string theories.

[1] We shall confine our argument in the left-moving sector. The right-moving sector can be treated in exactly same way.

2 2D black hole vs $c = 1$ string

Two-dimensional blackhole is constructed as a coset CFT of $SL(2,R)/U(1)$[10]. Here we skip how to define the theory and to derive the metric, but just start from the current algebra. The $SL(2,R)$ current algebra is defined as

$$J^0(y)J^0(z) \sim \frac{-k/2}{(y-z)^2}, \tag{4}$$

$$J^0(y)J^\pm(z) \sim \frac{\pm}{y-z}J^\pm(z), \tag{5}$$

$$J^+(y)J^-(z) \sim \frac{k}{(y-z)^2} - \frac{2}{y-z}J^0(z). \tag{6}$$

The energy-momentum tensor for $SL(2,R)$ part is given by the Sugawara form:

$$T_{SL(2,R)} = \frac{1}{k-2}\left[\frac{1}{2}\left(J^+J^- + J^-J^+\right) - J^0J^0\right], \tag{7}$$

where the central charge for this $T_{SL(2,R)}$ is $c = \frac{3k}{k-2}$. Gauging $U(1)$ is performed in standard way by first introducing the gauge current $\tilde{J}(z)$ which satisfies

$$\tilde{J}(y)\tilde{J}(z) \sim \frac{k/2}{(y-z)^2}, \tag{8}$$

and anti-commuting ghosts $\xi(z)$ and $\eta(z)$. Then defining the BRS charge for the diagonal part $J^0 + \tilde{J}$

$$Q_{U(1)} = \oint \frac{dz}{2\pi i}\eta(z)\left(J^0(z) + \tilde{J}(z)\right), \tag{9}$$

the coset $SL(2,R)/U(1)$ is given by the cohomology of $Q_{U(1)}$.

Now the total energy-momentum tensor for this system is given by

$$T_{tot} = T_{SL(2,R)} + \frac{1}{k}\tilde{J}\tilde{J} - \xi\partial\eta, \tag{10}$$

where the central charge is $c = \frac{3k}{k-2} + 1 - 2$ which equals 26 if $k = \frac{9}{4}$. According to the general procedure explained in the introduction, BRS charge Q_{diff} for the reparametrization (or diffeomorphism) is constructed with this T_{tot}. Then the physical spectrum is defined by the sum of both BRS charge of $U(1)$ and diffeomorphism:

$$Q = Q_{diff} + Q_{U(1)}, \tag{11}$$

$$Q|phys\rangle = 0. \tag{12}$$

Let us consider the following free field representation of the currents:

$$J^0 = \sqrt{\frac{k}{2}}\partial u, \tag{13}$$

$$J^\pm = i\left(\sqrt{\frac{k'}{2}}\partial\phi \pm i\sqrt{\frac{k}{2}}\partial X\right) e^{\pm i\sqrt{\frac{2}{k}}(X+iu)}, \tag{14}$$

$$\tilde{J} = -i\sqrt{\frac{k}{2}}\partial v, \tag{15}$$

where $k' = k-2$ and ϕ, X, u and v are free boson fields. With these fields T_{tot} is rewritten as

$$T_{tot} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{\sqrt{2k'}}\partial^2\phi - \frac{1}{2}(\partial X)^2 - \frac{1}{2}(\partial u)^2 - \frac{1}{2}(\partial v)^2 - \xi\partial\eta, \tag{16}$$

and BRS charge as

$$Q = \oint \frac{dz}{2\pi i}\left(cT_{tot} + bc\partial c\right) + \oint \frac{dz}{2\pi i}\sqrt{\frac{k}{2}}\eta(\partial u - i\partial v). \tag{17}$$

Note that first three terms in the free field representation of T_{tot} are the same expression of the energy-momentum tensor $T_{c=1}$ for flat $c=1$ string theory if we regard X and ϕ as matter and Liouville field respectively. In the BRS charge, however, reparametrization ghost and u, v, ξ and η are not decoupled, so we are not able to consider $c=1$ part and the rest separately.

In Ref.[11] we found the similarity transformation which brings everything into decoupled form of $c=1$ and the rest. The transformation is generated by the operator

$$R = \oint \frac{dz}{2\pi i}\frac{1}{\sqrt{2k}}c\xi(\partial u + i\partial v), \tag{18}$$

with which the BRS charge is transformed as

$$e^R Q e^{-R} = Q_{c=1} + Q_{U(1)} \tag{19}$$

$$= \oint \frac{dz}{2\pi i}\left(cT_{c=1} + bc\partial c\right) + \oint \frac{dz}{2\pi i}\sqrt{\frac{k}{2}}\eta(\partial u - i\partial v). \tag{20}$$

Here the BRS charge is decomposed into two totally decoupled parts: one is for the $c=1$ string and the other is for topological sector consists of u, v, ξ and η. With this form BRS cohomology becomes much simpler, i.e. just a direct product of independent cohomologies of $Q_{c=1}$ and $Q_{U(1)}$. Moreover, the cohomology of $Q_{U(1)}$ is trivial except for the zero mode of η; they are topological. Thus we have the total cohomology space

$$H^*_{SL(2,R)/U(1)} \simeq H^*_{c=1} \otimes H^*_{U(1)} = H^*_{c=1} \oplus \eta_0 H^*_{c=1}. \tag{21}$$

The $c=1$ string lives essentially in the flat background while the $SL(2,R)/U(1)$ string does in black hole, so this transformation relates the two apparently different background. Clarifying the meaning of the transformation should give new light on the background (in-) dependence of the string theory. We will try to give some hints toward this in later sections.

3 Polyakov's light-cone gauge

Quite similar structure as we saw in the previous section exists also in the relationship between conformal gauge and light-cone gauge *a la* Polyakov[12, 13] in the non-critical string theory. BRS quantization in the light-cone gauge is first discussed in Ref.[14, 15, 16] and refined later[17] to accommodate the so-called discrete states.

According to the Ref.[14] we start from the level k $SL(2,R)$ current algebra generated by $J^{\pm}$ and J^0. The Energy-momentum tensor for the gravity sector is given by the Sugawara form with improvement term:

$$T_{grav} = \frac{1}{k-2}\left[\frac{1}{2}\left(J^+J^- + J^-J^+\right) - J^0J^0\right] + \partial J^0. \tag{22}$$

We denote the energy-momentum tensor of the matter sector by T_m with its central charge c_m. The total energy-momentum tensor $T = T_m + T_{grav}$ and the current J^+ satisfy the following closed algebra,

$$\begin{aligned} T(y)J^+(z) &\sim \frac{1}{y-z}\partial J^+(z), && (23)\\ J^+(y)J^+(z) &\sim 0, && (24)\\ T(y)T(z) &\sim \frac{c/2}{(y-z)^4} + \frac{2}{(y-z)^2}T(z)\frac{1}{y-z}\partial T(z), && (25)\end{aligned}$$

where the central charge is given by $c = \frac{3k}{k-2} + 6k + c_m$.

Introducing ghost fields b, c for the generator T and ξ, η for J^+, BRS charge is defined by

$$Q_{l.c.} = \oint \frac{dz}{2\pi i}\left[c(T_m + T_{grav} + \partial\xi\eta) + bc\partial c\right] + \oint \frac{dz}{2\pi i}\eta J^+. \tag{26}$$

This time, we use Wakimoto's representation of $SL(2,R)$ current

$$\begin{aligned} J^+ &= \beta, && (27)\\ J^0 &= \beta\gamma - \sqrt{\frac{k'}{2}}\partial\phi, && (28)\\ J^- &= \beta\gamma^2 - k\partial\gamma - 2\sqrt{\frac{k'}{2}}\partial\phi\gamma, && (29)\end{aligned}$$

where β and γ are commuting ghosts and ϕ is free boson. In use of these, T_{grav} and $Q_{l.c.}$ are rewritten as

$$\begin{aligned} T_{grav} &= \partial\beta\gamma - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\alpha_+ + \frac{2}{\alpha_+})\partial^2\phi, \qquad (30) \\ Q_{l.c.} &= \oint \frac{dz}{2\pi i}\left[c\left(T_m - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\alpha_+ + \frac{2}{\alpha_+})\partial^2\phi\right) + bc\partial c + c\partial\beta\gamma + c\partial\xi\eta\right] \\ &\quad + \oint \frac{dz}{2\pi i}\eta\beta, \qquad (31) \end{aligned}$$

where $\alpha_+ = \sqrt{\frac{2}{k'}}$.

Besides two terms $c\partial\beta\gamma + c\partial\xi\eta$, $Q_{l.c.}$ is the sum of two independent parts, one of which is the same expression of the BRS charge in the conformal gauge if we regard ϕ as the Liouville field, and the other part is topological. Again we have similarity transformation[18] generated by[2]

$$R = \oint \frac{dz}{2\pi i}\left(-\gamma c\partial\xi\right). \qquad (32)$$

This eliminates just undesired terms and brings $Q_{l.c.}$ into the sum of BRS charge of conformal gauge and that of topological model, i.e. $e^R Q_{l.c.} e^{-R} = Q_{conf} + Q_{top}$. This establishes the relationship of both gauges[3].

4 Twisted $N = 2$ SCA in the topological sector

In the previous two examples, black hole and light-cone gauge, there is a common feature, i.e. the existence of topological sector. We have nice algebraic machinery in such topological systems: twisted $N = 2$ superconformal algebra

$$\begin{aligned} G^{\pm}(y)G^{\pm}(z) &\sim 0, \qquad (33) \\ G^{+}(y)G^{-}(z) &\sim \frac{d/3}{(y-z)^3} + \frac{1}{(y-z)^2}J(z) + \frac{1}{y-z}T_{top}(z), \qquad (34) \\ J(y)G^{\pm}(z) &\sim \frac{\pm 1}{y-z}G^{\pm}(z), \qquad (35) \\ J(y)J(z) &\sim \frac{d/3}{(y-z)^2}. \qquad (36) \end{aligned}$$

Actually, these generators can be expressed by the commuting ghosts β, γ and the anti-commuting ghosts ξ, η with a parameter λ as

$$G^{+} = \eta\beta, \qquad (37)$$

[2]This R operator already appeared in [14], although it was not recognized as the generator of the similarity transformation.

[3]See [19] for the treatment of the right-mover.

$$
\begin{aligned}
G^- &= \lambda\partial\xi\gamma + (\lambda-1)\xi\partial\gamma, && (38)\\
J &= \lambda\beta\gamma + (\lambda-1)\xi\eta, && (39)\\
T_{top} &= \lambda(\partial\beta\gamma + \partial\xi\eta) + (\lambda-1)(\beta\partial\gamma + \xi\partial\eta). && (40)
\end{aligned}
$$

where the central charge is given by $d = 3(1-2\lambda)$. The BRS charge of this topological system is expressed in terms of twisted $N=2$ generator as $Q_{top} = \oint \frac{dz}{2\pi i} G^+(z)$, so we can identify the fields and the parameter λ for the previous examples; for the light-cone gauge case $\lambda = 1$ and fields are literal, while black hole case $\lambda = 0$ and β, γ are identified with $\sqrt{k/2}\partial(u-iv)$, $1/\sqrt{2k}(u+iv)$ respectively.

In terms of this topological algebra the generator of similarity transformation R in both cases can be expressed in common way simply as

$$
R = \oint \frac{dz}{2\pi i}(-c(z)G^-(z)). \tag{41}
$$

Hence it turns out that the mechanism eliminating the coupling terms comes from the topological algebraic origin.

Generally, if we have two sectors with energy-momentum tensor $T_{c=26}$ and T_{top} whose central charges are $c = 26$ and $c = 0$ respectively, and the $c = 0$ part is governed by the twisted $N = 2$ SCA supplemented with the relation $G^-(y)T_{c=26}(z) \sim 0$, then BRS charge for this string

$$
Q = \oint \frac{dz}{2\pi i}\left[c(T_{c=26} + T_{top}) + bc\partial c\right] + \oint \frac{dz}{2\pi i} G^+ \tag{42}
$$

is transformed into totally decoupled form

$$
e^R Q e^{-R} = \oint \frac{dz}{2\pi i}\left[cT_{c=26} + bc\partial c\right] + \oint \frac{dz}{2\pi i} G^+ \tag{43}
$$

by the $R = \oint \frac{dz}{2\pi i}(-cG^-)$. We note here that the analogous structure is known in the topological string case[20].

5 G/H coset CFT coupled to 2D gravity

The previous argument can be generalized to G/H coset. Let us denote $J^A(z)$ as a current of G current algebra

$$
J^A(y)J^B(z) \sim \frac{k/2}{(y-z)^2}\delta^{AB} + \frac{if_G^{ABC}}{y-z}J^C(z), \tag{44}
$$

with energy-momentum tensor

$$T_G(z) = \frac{1}{k+h_G} J^A(z) J^A(z), \tag{45}$$

where h_G is defined by the relation $f_G^{ACD} f_G^{BCD} = h_G \delta^{AB}$.

Let $H^a(z)$ be a current associated with the subgroup H of G, for which energy-momentum tensor is denoted by $T_H(z)$. According to the standard procedure, G/H coset is constructed by gauging H part. We introduce gauge current $\tilde{H}^a(z)$ which satisfies same OPE as $H^a(z)$ but with the level $\tilde{k}$ defined by the relation $k + \tilde{k} + 2h_H = 0$, and a set of anti-commuting ghosts $\xi^a(z)$ and $\eta^a(z)$ with the OPE $\xi^a(y)\eta^b(z) \sim \frac{\delta^{ab}}{y-z}$. Then BRS charge

$$Q_H = \oint \frac{dz}{2\pi i} \left[\eta^a (H^a + \tilde{H}^a) - \frac{i}{2} f_H^{abc} \xi^a \eta^b \eta^c \right] \tag{46}$$

defines the G/H physical states. The total energy-momentum tensor is a sum of each for G current, $\tilde{H}$ current and ghosts: $T_{total} = T_G + T_{\tilde{H}} - \xi^a \partial \eta^a$. This expression can be rearranged into the sum of each for G/H and H/H

$$T_{total} = T_{G/H} + T_{H/H}, \tag{47}$$

where

$$T_{G/H} = T_G - T_H, \tag{48}$$

$$T_{H/H} = T_H + T_{\tilde{H}} - \xi^a \partial \eta^a. \tag{49}$$

The H/H part is topological as $T_{H/H}$ is Q_H exact[21]

$$T_{H/H} = \left\{ Q_H \, , \, \frac{1}{k+h_H} \xi^a (H^a - \tilde{H}^a) \right\}. \tag{50}$$

As a string theory this G/H matter couples to two-dimensional gravity, so the total BRS charge is the sum of Q_{diff} made of T_{total} and Q_H

$$Q = Q_{diff} + Q_H \tag{51}$$

$$= \oint \frac{dz}{2\pi i} \left[c T_{total} + bc\partial c \right] + Q_H. \tag{52}$$

Again the similarity transformation can be defined as before. That is to say the generator

$$R = \oint \frac{dz}{2\pi i} \frac{-1}{k+h_H} c \xi^a (H^a - \tilde{H}^a) \tag{53}$$

transforms BRS charge as

$$e^R Q e^{-R} = \oint \frac{dz}{2\pi i} \left[c T_{G/H} + bc\partial c \right] + Q_H, \tag{54}$$

so that it is separated into G/H string and H/H topological parts.

At this point a natural question arises; are there twisted $N = 2$ SCA also for this system? The answer is no, instead we have topological Kazama algebra[22], which is realized as follows:

$$\begin{aligned}
G^+ &= \eta^a(H^a + \tilde{H}^a) - \frac{i}{2} f_H^{abc} \xi^a \eta^b \eta^c, && (55)\\
G^- &= \frac{1}{k + h_H} \xi^a (H^a - \tilde{H}^a), && (56)\\
J &= -\xi^a \eta^a, && (57)\\
T_{top} &= \frac{1}{k + h_H} H^a H^a + \frac{1}{\tilde{k} + h_H} \tilde{H}^a \tilde{H}^a - \xi^a \partial \eta^a, && (58)\\
F &= \frac{-1}{2(k + h_H)^2} \left[h_H \xi^a \partial \xi^a + i f_H^{abc} \xi^a \xi^b (H^c + \tilde{H}^c) \right], && (59)\\
\Phi &= \frac{-1}{6(k + h_H)^2} i f_H^{abc} \xi^a \xi^b \xi^c. && (60)
\end{aligned}$$

This is essentially the same construction in Ref.[23]. Note that if H is abelian F and Φ disappear so that the algebra reduces to twisted N=2 SCA.

As in the twisted $N = 2$ case, the similarity transformation generator R is expressed as eq.(41) in terms of $G^-(z)$ in the Kazama algebra. However, the OPE G^-G^- does not vanish contrary to twisted $N = 2$ SCA, instead

$$G^-(y) G^-(z) \sim \frac{-2}{y - z} F(z). \tag{61}$$

Nevertheless, the mechanism still works for the separation of topological sector from the string theory. It is interesting to clarify to what extent this structure can be generalized into more general topological theory.[4]

6 $N = 0$ string as $N = 1$ string

In this section, we describe another example which includes supersymmetric generator. This is called Berkovits-Vafa[24] superstring which is a special $N = 1$ fermionic string equivalent

[4] In course of the symposium, it is pointed out by R. Stora that our similarity transformation is in parallel with the Kirkman map in the equivariant cohomology. I thank R. Stora for the comment.

to $N=0$ string. In a sense, it gives an example of a certain vacuum of $N=1$ string on which world sheet supersymmetry is broken to $N=0$.

We begin with an arbitrary $c=26$ energy-momentum tensor T_m and spin $(\frac{3}{2},-\frac{1}{2})$ fermionic ghosts b_1, c_1. They form a $c=15$ $N=1$ SCA as follows:

$$T_{N=1} = T_m - \frac{3}{2}b_1\partial c_1 - \frac{1}{2}\partial b_1 c_1 + \frac{1}{2}\partial^2(c_1\partial c_1), \tag{62}$$

$$G_{N=1} = b_1 + c_1(T_m + \partial c_1 b_1) + \frac{5}{2}\partial^2 c_1. \tag{63}$$

They satisfy the OPE

$$T_{N=1}(y)T_{N=1}(z) \sim \frac{15/2}{(y-z)^4} + \frac{2}{(y-z)^2}T_{N=1}(z) + \frac{1}{y-z}\partial T_{N=1}(z), \tag{64}$$

$$T_{N=1}(y)G_{N=1}(z) \sim \frac{3/2}{(y-z)^2}G_{N=1}(z) + \frac{1}{y-z}\partial G_{N=1}(z), \tag{65}$$

$$G_{N=1}(y)G_{N=1}(z) \sim \frac{10}{(y-z)^3} + \frac{2}{y-z}T_{N=1}(z). \tag{66}$$

BRS charge for this string is made up with these operators

$$Q_{N=1} = \oint \frac{dz}{2\pi i}\left(cT_{N=1} - \frac{1}{2}\gamma G_{N=1} + bc\partial c - \frac{1}{4}b\gamma^2 + \frac{1}{2}\partial c\beta\gamma - c\beta\partial\gamma\right), \tag{67}$$

where anti-commuting ghosts b, c are associated with the generator $T_{N=1}$ and commuting ghosts β, γ with $G_{N=1}$.

It seems to be complicated to prove with the expression (67) that this system is actually equivalent to the $N=0$ string. The following similarity transformation, however, makes things astonishingly simple[25]. That is to say, with

$$R = \oint \frac{dz}{2\pi i} c_1\left(\frac{1}{2}\gamma b - 3\partial c\beta - 2c\partial\beta - \frac{1}{2}\partial c_1 cb + \frac{1}{4}\beta\gamma\partial c_1\right), \tag{68}$$

BRS charge $Q_{N=1}$ is transformed into much simpler form: just a sum of each for $N=0$ string and topological system

$$e^R Q_{N=1} e^{-R} = Q_{N=0} + Q_{top}, \tag{69}$$

where

$$Q_{N=0} = \oint \frac{dz}{2\pi i}(cT_m + bc\partial c), \tag{70}$$

$$Q_{top} = \oint \frac{dz}{2\pi i}\left(-\frac{1}{2}b_1\gamma\right). \tag{71}$$

Moreover, the cohomology of Q_{top} is trivial, i.e. only a vacuum. Thus the cohomology of $Q_{N=1}$ is isomorphic to that of $Q_{N=0}$.

In this case, the identification of twisted $N=2$ generator in the operator R is not clear at first sight. The similarity transformation, however, can be decomposed into two steps one of which is actually expressed by the twisted $N=2$ generator. Namely, as the first step

$$R_1 = \oint \frac{dz}{2\pi i}\left(\frac{1}{2}c_1 b\gamma - \frac{1}{2}c_1\partial c_1\beta\gamma\right) \tag{72}$$

transforms BRS charge into simpler form which resembles former examples

$$e^{R_1}Q_{N=1}e^{-R_1} = \oint \frac{dz}{2\pi i}\left[c(T_m + T_{b_1c_1\beta\gamma} + bc\partial c\right] + \oint \frac{dz}{2\pi i}\left(-\frac{1}{2}b_1\gamma\right). \tag{73}$$

Then we can identify twisted $N=2$ generators as $G^+ = -\frac{1}{2}b_1\gamma$ and $G^- = c_1\partial\beta + 3\partial c_1\beta$. In terms of this, the second step is now familiar form

$$R_2 = \oint \frac{dz}{2\pi i}(-cG^-), \tag{74}$$

$$e^{R_2}\left(e^{R_1}Q_{N=1}e^{-R_1}\right)e^{-R_2} = Q_{N=0} + Q_{top}. \tag{75}$$

Thus in this way we can see again the role of the twisted $N=2$ algebra in the similarity transformation.

7 Non-linear realization

So far we have looked at the similarity transformation from the topological algebra point of view. There is another standpoint from which we can reinterpret the transformation, i.e. the non-linear realization of gauge symmetry.

Let us consider a finite dimensional Lie group G for the illustration of the idea[26]. Let T_a $(a = 1, \ldots, \dim(G))$ be a generator of G. Also we denote a generator of subgroup H by T_i $(i = 1, \ldots, \dim(H))$ and that of G/H coset by X_α $(\alpha = 1, \ldots, \dim(G) - \dim(H))$. Here we assume that G/H is symmetric

$$[T_i, T_j] = f_{ij}{}^k T_k, \tag{76}$$

$$[T_i, X_\alpha] = f_{i\alpha}{}^\beta X_\beta, \tag{77}$$

$$[X_\alpha, X_\beta] = f_{\alpha\beta}{}^k T_k. \tag{78}$$

G-algebra valued one-form

$$g^{-1}dg = \omega^a T_a \tag{79}$$

satisfies Maurer-Cartan equation

$$d\omega^a + \frac{1}{2} f_{bc}{}^a \omega^b \wedge \omega^c = 0. \tag{80}$$

This can be expressed by another parametrization, e.g. right coset parametrization

$$g = e^{y^i T_i} e^{\xi^\alpha X_\alpha}. \tag{81}$$

Denoting ϕ^i as a one-form on H-orbit defined by

$$e^{-y^i T_i} d e^{y^i T_i} = \phi^i(y) T_i \,, \tag{82}$$

one-form (79) is rewritten as

$$\begin{aligned} g^{-1}dg &= e^{-\xi^\alpha X_\alpha} \phi^i T_i e^{\xi^\alpha X_\alpha} + e^{-\xi^\alpha X_\alpha} d e^{\xi^\alpha X_\alpha} && (83) \\ &= \phi^i (T_i - \xi^\beta f_{\beta i}{}^\alpha X_\alpha + \cdots) + d\xi^\alpha (X_\alpha - \frac{1}{2} \xi^\beta f_{\beta\alpha}{}^i T_i + \cdots). && (84) \end{aligned}$$

Comparing this expression and (79), we obtain the transformation matrix $U(\xi)$ for basis change

$$(\omega^i \, \omega^\alpha) = (\phi^j \, d\xi^\beta) \, U^{-1}(\xi). \tag{85}$$

One the other hand, vector field Y_a on G satisfies

$$[Y_a, Y_b] = f_{ab}{}^c Y_c, \qquad \omega^a(Y_b) = \delta^a{}_b. \tag{86}$$

Corresponding to the right-coset parametrization, we also have vector field η_i on H-orbit

$$[\eta_i, \eta_j] = f_{ij}{}^k \eta_k, \qquad \phi^i(\eta_j) = \delta^i{}_j. \tag{87}$$

Then the basis change for the vector field is obtained by

$$\begin{pmatrix} Y_i \\ Y_\alpha \end{pmatrix} = U(\xi) \begin{pmatrix} \eta_j \\ \frac{\partial}{\partial \xi^\beta} \end{pmatrix}. \tag{88}$$

This relation was used to reproduce the non-linearly realized super-current $G_{N=1}$ in the previous section[27, 28], where anti-commuting field b_1 is nothing but the Nambu-Goldstone fermion associated to the broken generator $G_{N=1}$.

BRS charge Q_G is a corresponding object with the exterior derivative on G

$$d = \omega^a Y_a, \tag{89}$$

and defined by

$$Q_G = c^a \left(Y_a + \frac{1}{2} T^{gh}_a \right), \tag{90}$$

where $T^{gh}_a = -f_{ab}{}^c c^b b_c$, and the ghost variables satisfy $\{b_a, c^b\} = \delta_a{}^b$. For the right-coset basis $(\phi^i \, d\xi^\alpha)$ the exterior derivative is expressed as

$$d = \phi^i \eta_i + d\xi^\alpha \frac{\partial}{\partial \xi^\alpha}, \tag{91}$$

hence the corresponding BRS charge

$$\tilde{Q}_G = \tilde{c}^i \left(\eta_i - \frac{1}{2} f_{ij}{}^k \tilde{c}^j \tilde{b}_k \right) + \tilde{c}^\alpha \frac{\partial}{\partial \xi^\alpha}, \tag{92}$$

where the $\tilde{b}_a$ and $\tilde{c}^a$ are another set of ghosts satisfies $\{\tilde{b}_a, \tilde{c}^b\} = \delta_a{}^b$.

$\tilde{Q}_G$ should be obtained by the basis change (85) and (88) from Q_G. The basis change of ghost is thereby induced

$$c^a \;\rightarrow\; \tilde{c}^a = c^b (U^{-1})_b{}^a. \tag{93}$$

This can be achieved by the similarity transformation, namely

$$e^R c^a e^{-R} = c^b (U^{-1})_b{}^a, \tag{94}$$

where the operator R is defined by

$$R = c^a K_a{}^b b_b, \tag{95}$$

such that $(e^K)_b{}^a = (U^{-1})_b{}^a$.

These arguments can be generalized to the infinite dimensional case and also the case which includes fermionic generators. Actually the similarity transformation (68) in the previous section was able to be reproduced in this way[26].

Moreover, the transformations in the blackhole and light-cone gauge examples can also be reproduced in this context. For the 2D blackhole case, the starting algebra is a closed algebra generated by the currents T_{tot} and $J_{U(1)} = J^0 + \tilde{J}$. The $J_{U(1)}$ is non-linearly realized as $\sqrt{\frac{k}{2}} \partial(u - iv)$ and so $u - iv$ is Nambu-Goldstone (N-G) boson.

Similarly, in the light-cone gauge case, the algebra is generated by T and J^+. And $J^+ = \beta$ is broken generator, so that the β is N-G boson.

8 Summary

We have analyzed the various cases of the similarity transformations in terms of the topological algebra and of the non-linear realization of gauge symmetry. We have shown the universal structure behind the similarity transformation which may play an important role to understand the background (in-)dependence of the string theory.

It is, in a sense, natural to be able to understand the same phenomena from two different approaches, topological algebra and non-linear realization; they both describe the decoupling of gauge degrees of freedom.

Recently, the notion of duality is being drawn much attention in order to understand non-perturbative aspects of the string theory. In this regards, the discussion extended here may shed another light on the problems. For example, the T-duality can be understood by the gauge symmetry[29] which is always broken unless it is on the self-dual point. Then our argument in the previous section can be cast into the game. This will be reported elsewhere.

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